विद्या सर्वार्थि साधिका

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PERIODIC TEST – 2

Class: XII

Subject: Mathematics (041) M.M : 80

Date : 25 – 09 – 2023 Time : 3 Hours

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) -type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) -type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) -type questions of 5 marks each.
6. Section E has 3 case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions) Each question carries 1 mark.

1. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|adj\ A| =$ _____. (1)

(A) 0 (B) a^3 (C) a^2 (D) a^6

2. The value of $\tan^{-1}\left(\sin^{-1}\left(-\frac{\pi}{2}\right)\right) =$ _____. (1)

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{3}$ (C) $-\frac{\pi}{4}$ (D) $-\frac{\pi}{2}$

3. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} then\ A^2 =$ _____ (1)

(A) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ (B) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

(C) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$ (D) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

4. If A = $[1 \quad 2 \quad 3]$ then AA^T is of order _____. (1)

(A) 1×1 (B) 3×3 (C) 3×1 (D) 1×3

5. The value of k for which the following function is continuous at $x = 0$. (1)

$$f(x) = \begin{cases} k(x^2 + 2), & if\ x \leq 0 \\ 3x + 1, & if\ x > 0 \end{cases}$$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

6. If $\sqrt{x} + \sqrt{y} = 2$, then $\frac{dy}{dx} =$ _____ (1)

(A) $-\sqrt{\frac{y}{x}}$ (B) $\sqrt{\frac{y}{x}}$ (C) $-\sqrt{\frac{x}{y}}$ (D) 1

7. If $\int \frac{x^3}{\sqrt{1+x^2}} dx = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$, then find the values of $a\ and\ b$. (1)

(A) $a = \frac{1}{3},\ b = 1$ (B) $a = \frac{1}{3},\ b = -1$ (C) $a = 1,\ b = -1$ (D) $a = \frac{1}{3},\ b = \frac{1}{3}$

8. Volume of a cube is increasing at the rate of $7cm^3/sec$. How fast is the surface area increasing when the length of the edge is $12\ cm$. (1)

(A) $\frac{3}{7}cm^2/sec$ (B) $\frac{2}{3}cm^2/sec$ (C) $\frac{5}{3}cm^2/sec$ (D) $\frac{7}{3}cm^2/sec$

9. A stone is dropped in to a quiet lake and waves move in circles at a speed of $4cm$ per second. At the instant when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing? (1)

(A) $80\ cm^2/sec$ (B) $40\,\pi\ cm^2/sec$ (C) $80\,\pi\ cm^2/sec$ (D) $20\,\pi\ cm^2/sec$

10. The value of $\int_8^{13} \frac{\sqrt{21-x}}{\sqrt{x}+\sqrt{21-x}}\,dx$ = _____. (1)

(A) $\frac{5}{2}$ (B) $\frac{2}{5}$ (C) $\frac{13}{2}$ (D) $\frac{5}{13}$

11. If $x = 4t,\ \ y = \frac{4}{t}$ then $\frac{dy}{dx}$ = _____ (1)

(A) $\frac{1}{t^2}$ (B) t^2 (C) $-\frac{1}{t^2}$ (D) $-t^2$

12. If $y = cos(sinx^2)$, then $\frac{dy}{dx}\ at\ x = \sqrt{\frac{\pi}{2}}$ = _____. (1)

(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{1}{3}$

13. If a relation R on the set $A = \{a, e, i\}$ be defined by $R = \{(a, e)\}$then R is _____ (1)

(A) reflexive relation (B) symmetric relation
(C) transitive relation (D) Identity relation

14. If $y = ae^{mx} + be^{-mx}$ then $\frac{d^2y}{dx^2}$ = _____. (1)

(A) m^2y (B) $-m^2y$ (C) m^2 (D) my

15. Evaluate: $\int \frac{\sqrt{2+logx}}{x}dx$ (1)

(A) $\frac{2}{3}(2+logx)^{\frac{3}{2}}+c$ (B) $\frac{3}{2}(2+logx)^{\frac{3}{2}}+c$ (C) $(2+logx)^{\frac{3}{2}}+c$ (D) $\frac{1}{2}(2+logx)^{\frac{1}{2}}+c$

16. Evaluate: $\int \frac{sec^2x}{cosec^2x}dx$ (1)

(A) $cotx - x + C$ (B) $tanx + C$ (C) $tanx + x + C$ (D) $tanx - x + C$

17. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$ (1)

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{-\pi}{2}$ (D) $\frac{\pi}{6}$

18. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, then range of R is____. (1)

(A) $\{1, 3\}$ (B) $\{1, 2, 3, 4\}$ (C) $\{1, 2, 3\}$ (D) $\{1, 2, 3,4,5,6,7,8\}$

In the following question number 19 & 20 a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

19. Consider the function $f : R \to R$ defined as $f(x) = x^3$. (1)
Assertion (A): $f(x)$ is a one – one function.
Reason (R) : $f(x)$ is a one – one function if co- domain = range

20. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ (1)
Assertion (A) :$2\,A^{-1} = 9I - A$;
Reason (R) : $A^{-1} = \frac{1}{|A|}\,(adj\ A)$.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Evaluate : $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$. (2)

22. Evaluate : $\int cosx\,\sqrt{1-cos2x}\ dx$ (2)

OR

Evaluate : Evaluate: $\int \frac{dx}{x^2+2x+2}$

23. Determine the value of k for which the matrix: $A = \begin{bmatrix} 2 & -1 & k \\ 1 & -2 & 1 \\ 3 & 1 & -2 \end{bmatrix}$ is singular. (2)

24. Find the interval in which $f(x) = \frac{x}{logx}$ is increasing. (2)

OR

Show that the function $f(x) = 2 - 3x + 3x^2 - x^3$ is decreasing in R.

25. Show that the relation R in the set of real numbers, defined as $R = \{(a, b): a \leq b^2\}$ is neither reflexive nor symmetric. Justify with example. (2)

SECTION C

This section comprises of short answer type-questions (SA) of 3 marks each.

26. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^3 = a^3I + 3a^2bA$. (3)

OR

Find non-zero values of *x* satisfying the matrix equation :

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2 + 8 & 24 \\ 10 & 6x \end{bmatrix}$$

27. Show that the function f in A = $R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one – one and on -to. (3)

28. If $A = \begin{bmatrix} cos\alpha & -sin\alpha & 0 \\ sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, verify that $A\,.adjA = |A|I$ (3)

29. If $x = asec^3\theta$ and $y = atan^3\theta$ find $\frac{d^2y}{dx^2}$ at x = $\frac{\pi}{4}$. (3)

OR

If $y = e^{tan^{-1}x}$, *show that* $(1 + x^2)y_2 + (2x - 1)y_1 = 0$

30. Divide 4 into two positive numbers such that the sum of the squares of one number and the cube of the other number is minimum. (3)

31. Evaluate: $\int \frac{cosx}{(1-sinx)(2-sinx)}\,dx$. (3)

OR

Evaluate: $\int e^x\ cosx\,dx$.

SECTION C

This section comprises of Long Answer (LA) - type questions of 5 marks each.

32. Using matrix method, solve the following system of equations: (5)

$$2x + 3y + 3z = 5\ ;\quad x - 2y + z = -4\ ;\quad 3x - y - 2z = 3$$

33. *If* $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$, (5)
Prove that $(A + B)^2 \neq A^2 + B^2 + 2AB$.

34. If $x^4y^5 = (x + y)^9$, *prove that* $\frac{dy}{dx} = \frac{y}{x}$ (5)

OR

If $y = cot\,x + cosec\,x$, show that $sin\,x\left(\frac{d^2y}{dx^2}\right) = y^2$

35. Evaluate using properties of integration: $\int_0^{\pi} \frac{x}{1+sinx}\,dx$. (5)

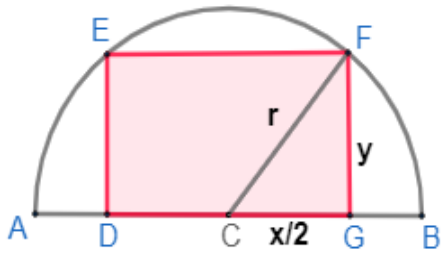
OR

Evaluate: $\int \frac{x+3}{\sqrt{5-4x+x^2}}\,dx$.

SECTION –E

This section comprises of 3 case- study/ passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.

36. A rectangle is inscribed in a semi- circle of radius r with one of its sides on the diameter of the semi- circle. Using the concept of maxima and minima, we need to find the dimensions of the rectangle, so that its area is maximum.



Use the figure to answer the following.

i) Find the area of rectangle A in terms of r and x. (1)

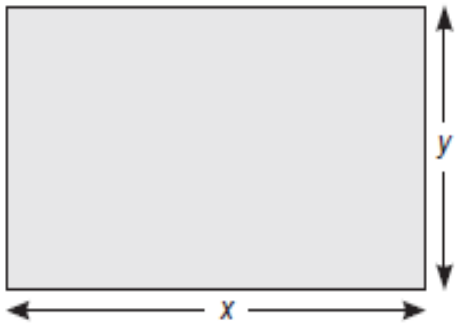
ii) The value of x in terms of r =____. (1)

iii) Find the length and breadth of the rectangle (x and y) in terms of r. (2)

OR

iii) Maximum area = _____

37. Aditi wants to donate a rectangular plot of land for a school in her village. When she was asked to give dimensions of the plot, she told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.



Based on the information given above, answer the following questions:

i) The equations in terms of x and y are _____& _____. (1)

ii) Which of the following matrix equation is represented by the given information? (1)

(A) $\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ (B) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-50\\-550\end{bmatrix}$

(C) $\begin{bmatrix}1 & 1\\2 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ (D) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$

iii) The value of x (length of rectangular field) is _____.

OR

iii) How much is the area of rectangular field? (2)

38. Mansi visited an exhibition along with her family. The exhibition had a huge swing, which attracted many children. Mansi found that the swing traced the path of a Parabola as given by $y = x^2$.

Answer the following questions using the above information.

i) Let $f : R \to R$ be defined by $f(x) = x^2$.
Check whether f is bijective or not. (2)

ii) Let $f : N \to N$ be defined by $f(x) = x^2$.
Show that f one – one. (2)